狀態轉換下原油期貨對非能源商品的交叉避險績效

The Cross Hedging Effectiveness of Oil Futures for Non-energy Commodities under Regime Switching

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投稿日期：2014年5月17日；第一次校訂：2015年1月24日；接受刊登日期：2015年3月18日

Received: May 17, 2014; First Revision: Jan. 24, 2015; Accepted: Mar. 18, 2015

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摘　要

本文提出一個同時使用原油期貨及相對應非能源商品期貨的交叉避險策略用來管理非能源商品現貨的價格風險。我們應用多重隨機係數自我迴歸馬可夫狀態轉換模型(*MRCARRS*)同時估計原油期貨及相對應非能源商品期貨的最適避險比例，文中亦同時建構一個較為精簡的部分狀態轉換*MRCARRS* (*PRCARRS*)進行多重期貨避險。

實證結果顯示在所有本文檢驗的非能源商品中，*MRCARRS*或*PRCARRS*避險策略有最佳的避險績效。根據Diebold、Mariano及West (*DMW*)的統計檢定量，多重期貨最小平方避險策略顯著不差於單一期貨最小平方避險策略，顯示多重期貨避險的優越性。此外，相較於本文其他競爭的避險策略，最佳避險策略(*MRCARRS*或*PRCARRS*)的*DMW*統計檢定量都為正值，顯示多變量狀態相依*RCARRS*模型有較優於狀態獨立及靜態避險模型的傾向。

關鍵詞：馬可夫狀態轉換、隨機係數自我迴歸模型、交叉避險、原油期貨、非能源商品

Abstract

This paper suggests a cross hedging strategy for managing non-energy commodity price risk using both crude oil futures and corresponded non-energy commodity futures. We apply multiple random coefficient autoregressive Markov regime switching models (*MRCARRS*) for simultaneously estimating the optimal hedge ratios of crude oil futures and non-energy commodity futures. We also envision a more parsimonious partial switching version of *MRCARRS* (*PRCARRS*) for multiple futures hedging.

Empirical results show that either *MRCARRS* or *PRCARRS* is the best performer for all commodities considered. According to the Diebold, Mariano and West (*DMW*) test statistics, the hedging performance of the multiple futures ordinary least square (*MOLS*) is statistically no worse than the single futures ordinary least square (*OLS*). This justifies the superiority of multiple futures hedging over single futures hedging. Moreover, all *DMW* statistics are positive for the best performer (*MRCARRS* or *PRCARRS*) over competing hedging strategies indicating that multivariate state-dependent *RCARRS* models have a tendency to outperform state-independent and static hedging models.

Keywords: Markov Regime Switching, Random Coefficient Autoregressive Model, Cross Hedging, Crude Oil Futures, Non-energy Commodities

I. Introduction

Crude oil is arguably the world's most important and actively-traded commodity and has a significant influence to most sectors of most economies because oil price shocks significantly affect real economic variables. A number of studies have investigated the effects of oil prices changes on real economic variables (Hutchison, 1993; Hamilton, 2003; Kilian, 2008; Gohin and Chantret, 2010). Other studies investigate the volatility transmission between crude oil market and equity markets (Geman and Kharoubi, 2008; Aloui and Jammazi, 2009; Gogineni, 2010; Jawadia et al., 2010; Arouri et al., 2011), the interactions between the crude oil market and other energy markets (Haigh and Holt, 2002; Ewing et al., 2002; Chen et al., 2005; Chang et al., 2010), and the comovement of crude oil market and exchange rate markets (Zhang et al., 2008; Harri et al., 2009). While there are a large body of studies investigates the volatility transmission in financial and energy markets, studies on the information content of crude oil and non-energy commodity markets are relatively few (Gohin and Chantret, 2010; Du et al., 2011; Serra, 2011; Ji and Fan, 2012). Furthermore, the literature on the effect of incorporating crude oil futures with non-energy commodity futures for hedging non-energy commodity holdings is also limited (Wu et al., 2011).

Due to increased use of biofuels in recent year, the comovement between the oil market and the agriculture market has become closer (Wu et al., 2011; Du et al., 2011). Du et al. (2011) find evidence of increasing volatility spillover among crude oil, corn, and wheat markets in recent year and conclude that the spillover is largely explained by tightened interdependence between crude oil and corn and wheat markets induced by ethanol production. Thus, the incremental gain of hedging non-energy commodity with both corresponded non-energy commodity futures and crude oil futures is worth further investigation. Wu et al. (2011) apply a trivariate BEKK-GARCH model to study the volatility spillover effects and cross hedging in corn and crude oil futures. Results find evidence of significant spillovers from crude oil prices to corn cash and futures prices, and based on this strong volatility link between crude oil and corn prices, Wu et al. suggest a cross hedging strategy that uses both corn futures and crude oil futures to hedge the underlying corn spot holdings and find only slightly better hedging performance compared with traditional hedging in corn futures markets alone. Wu et al.'s findings, however, are based on the assumption that the state of the market conditions do not change over time.

In a series paper of Sarno and Valente (2000, 2005a, 2005b), the joint distribution between spot and futures returns has been justified to be affected by the "state of the market". Alizadeh and Nomikos (2004) pioneers the study of using regime switching model for implementing hedging strategy. The rationale behind this stems from the fact that the joint distribution between spot and futures returns may be characterized by regime shifts, which, in turn, suggests that in order to improve the futures hedging effectiveness, we might have to take account of this state-dependent property in estimating more efficient regime switching hedge ratios. In this line of research, a variety of more sophisticated regime switching models have been proposed to investigate the effects of regime switching on futures hedging (Lee et al., 2006;

Lee and Yoder, 2007; Alizadeh et al., 2008; Lee, 2009; Lee, 2010). Empirical evidence shows that incorporating regime switching effects improve futures hedging effectiveness. All of these models, however, are implemented for single futures hedging strategy. In this paper, we attempt to investigate the incremental gain of hedging non-energy commodity with both corresponded non-energy commodity futures and crude oil futures under regime switching.

Two general approaches have been applied to estimate the time-varying minimum variance hedge ratios (Lee et al., 2006). One approach is to estimate the hedge ratios by estimating the conditional second moments via a variety of GARCH models (Baillie and Myers, 1991; Kroner and Sultan, 1993; Gagnon and Lypny, 1995; Brooks et al., 2002; Lee and Yoder, 2007; Alizadeh et al., 2008; Lee, 2009; Lee, 2010, Wu et al., 2011). The other general approach treats the hedge ratio as a time varying coefficient and estimates the coefficient directly (Bera et al., 1997; Alizadeh and Nomikos, 2004; Lee et al., 2006). In this paper, we suggest a multiple random coefficient autoregressive Markov regime switching models (*MRCARRS*) model to investigate the effectiveness of crude oil futures and non-energy commodity futures for hedging non-energy commodity price risk under regime switching.

There are two main reasons that we apply *MRCARRS* for multiple futures hedging. First, *MRCARRS* encompasses many observed time series properties of hedge ratio dynamic and nests within it many previous hedging models. *MRCARRS* models the equilibrium time path of multiple hedge ratios simultaneously and also allows the hedge ratios to be dependent upon the state of the market. It nests the state independent multiple random coefficient autoregressive (*MRCAR*) model, the partial switching *MRCARRS* model (*PRCARRS*) and the traditional multiple ordinary least square hedging strategy (*MOLS*). Second, so far in this line of research, most of the existing regime-switching GARCH hedging models are bivariate because a full-switching trivariate regime switching GARCH model using both corresponded non-energy commodity futures and crude oil futures for hedging underlying spot might be subject to problems of overparameter and convergence (Hass et. al., 2004). A regime switching time-varying coefficient model like *MRCARRS* is relatively more parsimonious to implement than trivariate state-dependent GARCH models.

The remainder of the article is organized as follows. The multiple random coefficient autoregressive Markov regime switching (*MRCARRS*) model for multiple futures hedging is presented in section II. Section III gives minimum variance hedge ratio (MVHR) and measurements of hedging performance. This is followed by discussions of empirical results. A conclusion ends the article.

II. The Multiple Random Coefficient Autoregressive Regime Switching (*MRCARRS*) Model

The random coefficient autoregressive Markov regime switching (*RCARRS*) model is proposed by Lee et al. (2006) for single futures hedging strategy. In this paper, we further extend the model to multiple futures hedging strategy and the model is called the multiple random coefficient autoregressive Markov regime switching (*MRCARRS*) model. *MRCARRS* not only conditions the multiple hedge ratios on the state of market volatility but also characterizes the equilibrium time path of the multiple hedge ratios. In *MRCARRS*, we allow both the hedge ratios of oil futures and non-energy commodity futures to follow a regime switching random coefficient autoregressive process. The model is specified as

$$R_{c,t} = \alpha_{s_t} + \beta_t R_{f,t} + \gamma_t R_{o,t} + \varepsilon_{t,s_t}, \tag{1}$$

where $R_{c,t}$, $R_{f,t}$ and $R_{o,t}$ are the spot (cash) returns, futures returns and oil futures returns for time period t, respectively, and ε_{t,s_t} are independent and identically distributed (iid) random disturbances with mean zero and variance $\sigma^2_{\varepsilon,s_t}$. The unobserved state variable, S_t, follows a two-state, first-order Markov-switching process, with the following transition probabilities:

$$p(s_t = 1 \mid s_{t-1} = 1) = p = \frac{\exp(p_0)}{1+\exp(p_0)}, \tag{2}$$

$$p(s_t = 2 \mid s_{t-1} = 2) = q = \frac{\exp(q_0)}{1+\exp(q_0)}, \tag{3}$$

where p_0 and q_0 are unconstrained parameters estimated along with unknown system parameters. The hedge ratios β_t and γ_t are treated as latent variables following mean reverting processes given by

$$(\beta_t - \overline{\beta}) = \phi_{\beta,s_t} (\beta_{t-1} - \overline{\beta}) + v_{t,s_t}, \tag{4}$$

$$(\gamma_t - \overline{\beta}) = \phi_{\gamma,s_t}(\gamma_{t-1} - \overline{\gamma}) + u_{t,s_t} . \tag{5}$$

The state-dependent disturbances ε_{t,s_t} , v_{t,s_t} and u_{t,s_t} are assumed to be jointly normally distributed given by

$$\begin{bmatrix} \varepsilon_{t,s_t} \\ v_{t,s_t} \\ u_{t,s_t} \end{bmatrix} \sim N\left(\mathbf{0}, \begin{bmatrix} \sigma^2_{\varepsilon,s_t} & 0 & 0 \\ 0 & \sigma^2_{v,s_t} & 0 \\ 0 & 0 & \sigma^2_{u,s_t} \end{bmatrix} \right), \tag{6}$$

where $\mathbf{0}$ is a 3×1 zero vector. To estimate the *MRCARRS* model with Kim's filter (Kim, 1994), we have to convert *MRCARRS* into a standard state space form. We first replace the latent variables β_t and γ_t with $\delta_t = \beta_t - \overline{\beta}$ and $\tau_t = \gamma_t - \overline{\gamma}$, respectively and rewrite equations (1), (4) and (5) respectively as

$$R_{c,t} = w_{s_t} + \mathbf{R}_{F,t}\boldsymbol{\delta}_t + \varepsilon_{t,s_t} , \tag{7}$$

where $w_{s_t} = \alpha_{s_t} + \overline{\beta}R_{f,t} + \overline{\gamma}\, R_{o,t}$, $\mathbf{R}_{F,t} = \begin{bmatrix} R_{f,t} & R_{o,t} \end{bmatrix}$ is the futures returns vector and the vector of latent variables $\boldsymbol{\delta}_t = \begin{bmatrix} \delta_t & \tau_t \end{bmatrix}'$ has an error covariance matrix denoted as $\mathbf{M}$ with "$'$" standing for transpose. The dynamic of latent variables δ_t and τ_t are given by

$$\delta_t = \phi_{\beta,s_t}\, \delta_{t-1} + v_{t,s_t} , \tag{8}$$

$$\tau_t = \phi_{\gamma,s_t}\, \tau_{t-1} + u_{t,s_t} . \tag{9}$$

Equation (7), (8) and (9) constitute our *MRCARRS* model in state space form. Compared to Lee et al.'s (2006) hedging model, this hedging model allows all parameters to be state-dependent and it contains multiple futures contracts. *MRCARRS* is estimated with Kim's filter, an interleaving filter of Kalman filter and the Hamilton filter.[1] The estimation procedure can be summarized as follows. In the first step, run the Kalman filter given in equations (10)-(15) for $i, j = 1{,}2$:

<u>Prediction equations:</u>

$$\hat{\boldsymbol{\delta}}^{(i,j)}_{t|t\text{-}1} = \boldsymbol{\varphi}_j \, \hat{\boldsymbol{\delta}}^{i}_{t-1|t\text{-}1} \tag{10}$$

$$\mathbf{M}^{(i,j)}_{t|t\text{-}1} = \boldsymbol{\varphi}_j \, \mathbf{M}^{i}_{t-1|t\text{-}1} \boldsymbol{\varphi}'_j + \boldsymbol{\sigma}^2_{\mathrm{v},j} \tag{11}$$

where $\boldsymbol{\varphi}_j = \begin{bmatrix} \phi_{\beta,j} & 0 \\ 0 & \phi_{\gamma,j} \end{bmatrix}$ and $\boldsymbol{\sigma}^2_{\mathrm{v},j} = \begin{bmatrix} \sigma^2_{v,j} & 0 \\ 0 & \sigma^2_{u,j} \end{bmatrix}$.

<u>Updating equations:</u>

$$\eta^{(i,j)}_{t|t\text{-}1} = R_{c,t} - w_j - \mathbf{R}_{F,t} \hat{\boldsymbol{\delta}}^{(i,j)}_{t|t\text{-}1} \tag{12}$$

$$f^{(i,j)}_{t|t\text{-}1} = \mathbf{R}_{F,t} \mathbf{M}^{(i,j)}_{t|t\text{-}1} \mathbf{R}'_{F,t} + \sigma^2_{\varepsilon,j} \tag{13}$$

$$\hat{\delta}^{(i,j)}_{t|t} = \hat{\delta}^{(i,j)}_{t|t\text{-}1} + \mathrm{M}^{(i,j)}_{t|t\text{-}1} \mathrm{R}'_{F,t} (\mathrm{f}^{(i,j)}_{t|t-1})^{-1} \eta^{(i,j)}_{t|t\text{-}1} \tag{14}$$

[1] To estimate the proposed *MRCARRS* model, Kim's filter is applied to extract the latent hedge ratio sequence and regime probability simultaneously for completing the likelihood function. Kim's filter (Kim, 1994) is an interleaving filter of Kalman filter and the Hamilton filter. Kalman filter is applied to extract the latent hedge ratio sequence and Hamilton filter is applied to extract the regime probability.

$$\mathbf{M}_{t|t}^{(i,j)} = [\mathbf{I} - \mathbf{M}_{t|t-1}^{(i,j)} \mathbf{R}'_{F,t} (\mathrm{f}_{t|t-1}^{(i,j)})^{-1} \mathbf{R}_{F,t}]\ \mathbf{M}_{t|t\text{-}1}^{(i,j)} \tag{15}$$

where $\mathbf{I}$ is a 2×2 identity matrix, $\hat{\boldsymbol{\delta}}_{t-1|t-1}^{(i,j)}$ is the estimate of $\boldsymbol{\delta}_{t-1}$ and $\hat{\boldsymbol{\delta}}_{t|t\text{-}1}^{(i,j)}$ is a prior estimate of $\boldsymbol{\delta}_t$ based on information up to time $t-1$, given $s_{t-1} = i$ and $s_t = j$. $\mathbf{M}_{t|t-1}^{(i,j)}$ is the error covariance matrix of $\boldsymbol{\delta}_{t|t\text{-}1}^{(i,j)}$, $\eta_{t|t\text{-}1}^{(i,j)}$ is the conditional forecast error of $R_{c,t}$, and $f_{t|t\text{-}1}^{(i,j)}$ is the conditional variance of the forecast error $\eta_{t|t\text{-}1}$ based on the information up to time $t-1$ given $s_{t-1} = i$ and $s_t = j$.

The second step of Kim's filter is to calculate the regime probabilities $P(s_{t-1} | \psi_t)$ and $P(s_t | \psi_t)$ via Hamilton filter (Hamilton, 1989; Hamilton and Susmel, 1994) with the following steps:

Calculate $P(\mathrm{s}_\mathrm{t} | \psi_{t-1})$:

$$P(s_t = j, s_{t-1} = i | \psi_{t-1}) = P(s_t = j | s_{t-1} = i) P(s_{t-1} = i | \psi_{t-1}) \tag{16}$$

$$P(s_t = j | \psi_{t-1}) = \sum_{i=1}^{2} P(s_t = j | s_{t-1} = i) P(s_{t-1} = i | \psi_{t-1}) \tag{17}$$

Calculate $f(R_{\mathrm{s,t}} | \psi_{t-1})$:

$$\begin{aligned} & f(R_{\mathrm{c,t}} | s_t = j, s_{t-1} = i, \psi_{t-1}) \\ & \quad = (2\pi)^{-\frac{1}{2}} \left| f_{t|t-1}^{(i,j)} \right|^{-\frac{1}{2}} \exp\left(-\frac{1}{2} \eta_{t|t-1}^{(i,j)'} f_{t|t-1}^{(i,j)-1} \eta_{t|t-1}^{(i,j)} \right) \end{aligned} \tag{18}$$

$$f(R_{\mathrm{c,t}} | \psi_{t-1}) = \sum_{j=1}^{2} \sum_{i=1}^{2} f(R_{c,t} | s_t = j, s_{t\text{-}1} = i, \psi_{t-1}) P(s_t = j, s_{t\text{-}1} = i | \psi_{t-1}) \tag{19}$$

Update $P(s_t | \psi_t)$:

$$P(s_t = j, s_{t-1} = i | \psi_t) = \frac{f(R_{c,t}, s_t = j, s_{t-1} = i | \psi_{t-1})}{f(R_{c,t} | \psi_{t-1})} \tag{20}$$

$$= \frac{f(R_{c,t} | s_t = j, s_{t-1} = i, \psi_{t-1}) P(s_t = j, s_{t-1} = i | \psi_{t-1})}{f(R_{c,t} | \psi_{t-1})} \tag{21}$$

$$P(s_t = j | \psi_t) = \sum_{i=1}^{2} P(s_t = j, s_{t-1} = i | \psi_t), \tag{22}$$

where ψ_t and ψ_{t-1} refer to the information available at time t and $t-1$, respectively. The Hamilton filter is initiated with the steady-state probabilities of s_t give by

$$P(s_0 = 1 | \psi_0) = \frac{1-q}{2-p-q}, \tag{23}$$

$$P(s_0 = 2 | \psi_0) = \frac{1-p}{2-p-q}, \tag{24}$$

where p and q are defined in equation (2) and (3). The recursive nature of the regime switching Kalman filter (Kim's filter) produces a 2-fold increase in the number of cases to consider in each iteration of the filtering process that makes the model intractable. To make the evolution of the process tractable, in the third step of Kim's filter, we collapse (14) and (15) based on conditional expectations to mitigate parameter proliferation. In particular, $\hat{\boldsymbol{\delta}}^j_{t|t}$ and $\mathbf{M}^j_{t|t}$ are defined by

$$\hat{\delta}^{j}_{t|t} = \frac{\Sigma_{i=1}^{2} P(s_{t-1} = i, s_t = j \mid \psi_t)\hat{\delta}^{(i,j)}_{t|t}}{P(s_t = j \mid \psi_t)}, \text{ and} \tag{25}$$

$$\mathrm{M}^{j}_{t|t} = \frac{\Sigma_{i=1}^{2} P(s_{t-1} = i, s_t = j \mid \psi_t)\left[\mathrm{M}^{(i,j)}_{t|t} + \left(\hat{\delta}^{j}_{t|t} - \hat{\delta}^{(i,j)}_{t|t}\right)\left(\hat{\delta}^{j}_{t|t} - \hat{\delta}^{(i,j)}_{t|t}\right)^{T}\right]}{P(s_t = j \mid \psi_t)} \tag{26}$$

The vector $\hat{\boldsymbol{\delta}}^{j}_{t|t} = \left[\hat{\delta}_{t|t} \quad \hat{\tau}_{t|t}\right]^{T}$ is the estimate of $\boldsymbol{\delta}_t$ based on information up to time t, given $s_t = j$ and $\mathbf{M}^{j}_{t|t}$ is the error covariance matrix of $\hat{\boldsymbol{\delta}}^{j}_{t|t}$ based on information up to time t, given $s_t = j$.

The unknown parameters in *MRCARRS* with multiple futures contracts are $\boldsymbol{\Theta} = \{p_0\,,\ q_0\,,\ \alpha_{s_t}\,,\ \sigma^2_{\varepsilon,s_t}\,,\ \phi_{\beta,s_t}\,,\ \overline{\beta}\,,\ \sigma^2_{v,s_t}\,,\ \phi_{\gamma,s_t}\,,\ \overline{\gamma}\,,\ \sigma^2_{u,s_t}\}$ for $s_t = \{1,2\}$, which can be estimated by maximizing the following log-likelihood function with respect to the unknown parameters:

$$L(\Theta) = \sum_{t=1}^{T} \log f\left(R_{c,t} \mid \psi_{t-1}\right), \tag{27}$$

where T is the total number of observations and $f\left(R_{c,t} \mid \psi_{t-1}\right)$ is defined in (19).

Since our original attempt is to estimate $\hat{\beta}_{t|t}$ and $\hat{\gamma}_{t|t}$, we have to recover these estimates from the extracted $\hat{\delta}_{t|t}$ and $\hat{\tau}_{t|t}$ as follows:

$$\hat{\beta}^{j}_{t|t} = \hat{E}\left[\beta_t \mid s_{t-1} = i, s_t = j, \psi_t\right] = \hat{\delta}^{j}_{t|t} + \hat{\overline{\beta}}, \tag{28}$$

$$\hat{\gamma}^{j}_{t|t} = \hat{E}\left[\gamma_t \mid s_{t-1} = i, s_t = j, \psi_t\right] = \hat{\tau}^{j}_{t|t} + \hat{\bar{\gamma}}\,, \tag{29}$$

and the estimates of expected optimal hedge ratios for non-energy commodity futures and crude oil futures are respectively given by

$$\hat{\beta}^{*}_{t|t} = \hat{\beta}^{1}_{t|t} P(s_t = 1 \mid \psi_t) + \hat{\beta}^{2}_{t|t} P(s_t = 2 \mid \psi_t)\,, \tag{30}$$

$$\hat{\gamma}^{*}_{t|t} = \hat{\gamma}^{1}_{t|t} P(s_t = 1 \mid \psi_t) + \hat{\gamma}^{2}_{t|t} P(s_t = 2 \mid \psi_t)\,, \tag{31}$$

The out-of-sample one-step-ahead hedge ratios can be calculated as the weighted average of one-step-ahead forecasts of β_i and γ_i at time $t-1$ for $i = 1,2$. The one-step-ahead forecasts of β_i and γ_i at time $t-1$ are respectively defined as

$$\hat{\beta}^{j}_{t|t\text{-}1} = \hat{E}\left[\beta_t \mid s_{t-1} = i, \psi_{t-1}\right] = \hat{\delta}^{i}_{t|t\text{-}1} + \hat{\bar{\beta}}\,, \tag{32}$$

$$\hat{\gamma}^{j}_{t|t\text{-}1} = \hat{E}\left[\gamma_t \mid s_{t-1} = i, \psi_{t-1}\right] = \hat{\tau}_{t|t-1} + \hat{\bar{\gamma}}\,. \tag{33}$$

The one-step-ahead forecasts of hedge ratios can then be calculated as

$$\hat{\beta}^{*}_{t|t-1} = \hat{\beta}^{1}_{t|t-1} P(s_{t-1} = 1 \mid \psi_{t-1}) + \hat{\beta}^{2}_{t|t-1} P(s_{t-1} = 2 \mid \psi_{t-1})\,, \tag{34}$$

$$\hat{\gamma}^{*}_{t|t-1} = \hat{\gamma}^{1}_{t|t-1} P(s_{t-1} = 1 \mid \psi_{t-1}) + \hat{\gamma}^{2}_{t|t} P(s_{t-1} = 2 \mid \psi_{t-1})\,. \tag{35}$$

MRCARRS is a full switching model such that all system parameters are subject to regime shifting. Lee et al. (2006) find that a partial switching model might has superior out-of-sample hedging performance compared to its full switching counterpart. In this paper we also envision a partial switching *MRCARRS* model such that all system parameters in the transition equations are state independent. The partial switching *MRCARRS* model is denoted as *PRCARRS*.[2]

In this paper, we compare the hedging performance of *MRCARRS* with its nested models, the state independent multiple random coefficient autoregressive (*MRCAR*) model and the partial switching *MRCARRS* model (*PRCARRS*). In additions, the hedging performance of *MRCARRS* is also compared with the conventional ordinary least square hedging strategy (*OLS*) using only the corresponded non-energy commodity futures and the multiple regression hedging strategy (*MOLS*) using both corresponded non-energy commodity futures and crude oil futures.

III. Minimum Variance Hedge Ratio (MVHR) and Measurements of Hedging Performance

A risk-minimized hedger chooses a hedging strategy to minimize

2 In the partial switching *MRCARRS* model (*PRCARRS*), all system parameters in the transition equation are state independent. Namely, $(\beta_t - \overline{\beta}) = \phi_\beta (\beta_{t-1} - \overline{\beta}) + v_t$ and $(\gamma_t - \overline{\beta}) = \phi_\gamma (\gamma_{t-1} - \overline{\gamma}) + u_t \cdot$ Only those parameters in the measurement equation are subject to regime switching. If we further make all system parameters in the measurement equation to be state independent, we have the multiple random coefficient autoregressive (*MRCAR*) model (Bera et al., 1997). Both *MRCAR* and *PRCARRS* models are nested within the *MRCARRS* model.

the variance of the hedged portfolio return or equivalently to maximize the variance reduction of a hedging strategy compared to the unhedged position. The variance of the hedged portfolio with multiple futures contracts is equal to

$$Var\left(R_{c,t} - \hat{\beta}_t^* R_{f,t} - \hat{\gamma}_t^* R_{o,t}\right), \tag{36}$$

where $\hat{\beta}_t^*$ and $\hat{\gamma}_t^*$ are respectively the estimated hedge ratios of corresponded non-energy commodity futures and crude oil futures derived from alternative models. The percentage variance reduction (hedging effectiveness, HE) is defined as

$$HE = \frac{Var\left(R_{c,t}\right) - Var\left(R_{c,t} - \hat{\beta}_t^* R_{f,t} - \hat{\gamma}_t^* R_{o,t}\right)}{Var\left(R_{c,t}\right)} \times 100\,, \tag{37}$$

In addition to variance reduction, we also consider the economic significance of the superiority of *MRCARRS* over alternative models measured with utility functions. Consider a hedger with a mean-variance expected utility function (Kroner and Sultan, 1993; Lafuente and Novales, 2003; Alizadeh and Nomikos, 2004; and Lee et al., 2006, 2010):

$$E\left[U\left(R_{p,t}\right) | \psi_{t-1}\right] = E\left[R_{p,t} | \psi_{t-1}\right] - \kappa\, Var\left(R_{p,t} | \psi_{t-1}\right), \tag{38}$$

where κ is the coefficient of absolute risk aversion, E stands for expectation operator and $R_{p,t}$ is the return of hedged portfolio.

To further investigate the statistical significance of the superiority of *MRCARRS*, Diebold and Mariano (1995) and West (1996) (*DMW*) test statistics are applied. To construct the *DMW* statistic, let $\hat{d}_t = f(v_{A,t+1}) - f(v_{B,t+1})$, where $\bar{d} = N^{-1}\sum_{t=R+1}^{T}\hat{d}_{t+1}$, R denotes the length of estimation period, N is the length of the prediction period, T is the sample size, f is the square error loss function and $v_{A,t} = R_{c,t} - \hat{\beta}^{*}_{\mathrm{A},t}R_{f,t} - \hat{\gamma}^{*}_{\mathrm{A}\,t}R_{o,t}$ and $v_{B,t} = R_{c,t} - \hat{\beta}^{*}_{\mathrm{B},t}R_{f,t} - \hat{\gamma}^{*}_{\mathrm{B}\,t}R_{o,t}$. $\hat{\beta}^{*}_{\mathrm{A},t}$ and $\hat{\gamma}^{*}_{\mathrm{A}\,t}$ are respectively the hedge ratios of corresponded non-energy commodity futures and crude oil futures estimated from model A and $\hat{\beta}^{*}_{\mathrm{B},t}$ and $\hat{\gamma}^{*}_{\mathrm{B}\,t}$ are respectively the hedge ratios of corresponded non-energy commodity futures and crude oil futures estimated from model B. The *DMW* test statistic is computed as follows,

$$DMW = \frac{\bar{d}}{\sqrt{N^{-1}\,\hat{V}}}, \tag{39}$$

where $\hat{V} = N^{-1}\sum_{t=R+1}^{T}\left(\hat{d}_{t+1} - \bar{d}\right)^2$. For nested model, the critical values of *DMW* test have to be adjusted to produce correct tests (McCracken, 2007). The test is one-sided with the null hypothesis that the predictive ability of model A is not superior to model B which is given by

$$H_0 = E\left[f\left(v_{B,t+1}\right) - f\left(v_{A,t+1}\right)\right] \le 0, \tag{40}$$

while the alternative is

$$H_A = E\left[f\left(v_{B,t+1}\right) - f\left(v_{A,t+1}\right)\right] > 0. \tag{41}$$

Rejection of the null hypothesis implies that the predictive ability of model A is superior to model B.

IV. Data, Estimation Results, Hedging Effectiveness and Robustness Analysis

The proposed multiple futures cross hedging strategy is applied to nearby futures contracts of crude oil, platinum and palladium traded in the New York Mercantile Exchange (NYMEX), corn and wheat traded in the Chicago Board of Trade (CBOT) and coffee and sugar traded in the New York Board of Trade (NYBOT) for the period of January 1996 to December 2010. Spot and futures prices are Wednesday prices obtained from Datastream. The returns of each price series are computed as the changes in the natural logarithms of prices multiplied by 100. Estimation of all models was conducted using data from January 1996 to December 2008; the remaining data are used for out-of-sample analysis.

Table 1 summarize the estimation procedure for *MRCARRS*. The original model is transformed into standard state-space form to apply Kim's filter for estimation and then converted to original form for estimating hedge ratios of corresponded non-energy commodity futures and crude oil futures contracts. Table 2 presents the parameter estimation results of *MRCAR*, *PRCARRS* and *MRCARRS* models. We derive these results by maximizing the log-likelihood function in equation (27) with respect to the unknown parameters using the numerical constrained optimization (CO) procedure in GAUSS. From the measurement equation we find that most of the conditional means (α) are not significant. This simply shows the fact that hedged portfolio normally has a mean return close to zero. For those statistically significant conditional means, we find that high volatility state (high σ_ε) normally associates with a negative conditional mean and low volatility state normally associates with a positive conditional mean. This implies that during the market turmoil the mean returns on hedged portfolio tend to be negative and when the market is relatively tranquil, the mean returns on hedged portfolio tend to be positive. In the transition equation, $\bar{\beta}$ and $\bar{\gamma}$ stand for the steady state hedge ratios of corresponded non-energy commodity futures and crude oil futures, respectively. The steady state hedge ratio of corresponded non-energy commodity futures has a highest value of 0.994 for the case of corn estimated with *MRCARRS* and the steady state hedge ratio for crude oil futures has a highest value of 0.035 for sugar estimated with *MRCAR*. $\bar{\beta}$ is consistently higher than $\bar{\gamma}$ due to the higher correlation between corresponded non-energy commodity futures and its underlying. The volatility in the transition equations shows the flexibility of hedge ratios. Hedge ratio with higher volatility in the transition equations shows larger range of fluctuation. In general, we find that the volatility in the transition equations of corresponded non-energy commodity futures is consistently higher than that of crude oil futures. This implies that the hedge ratio of corresponded non-energy commodity futures is more volatile than the hedge ratio of crude oil futures.

In- and out-of-sample hedging effectiveness of alternative models are shown in Table 3. Take coffee data for instance, in-sample, the unhedged cash position has a variance of 14.049. If we use *OLS* hedging strategy, the variance of hedged portfolio has a variance of 6.442

or a variance reduction of 54.15% compared to unhedged position. If we adopt *MOLS* hedging strategy with both coffee futures and crude oil futures, the variance of the hedged portfolio return is equal to 6.437 or the variance reduction is equal to 54.18%. Both *OLS* and *MOLS* hedging are static hedging. If we adopt *MRCAR* hedging, the hedge ratio will be time-varying and hedgers can rebalance their positions based on the predicted hedge ratios for coffee futures and crude oil futures. The variance of the hedged portfolio return is equal to 6.600 or the variance reduction is equal to 53.02% if we use *MRCAR* hedging. Time-varying *MRCAR* hedging strategy does not create hedging gains for the case of coffee. If we adopt partial switching *PRCARRS* hedging strategy which limits only the measurement equation to be to be state dependent, the variance of the hedged portfolio return is equal to 5.866 or the variance reduction is equal to 58.24%. *PRCARRS* hedging strategy is superior to both static *OLS* and *MOLS* and the time-varying *MRCAR* hedging strategies.

The best in-sample performer for the case of coffee is the full switching *MRCARRS* which allows both measurement equation and transition equation to be state-dependent. *MRCARRS* has a hedged portfolio variance of 5.798 or a variance reduction of 58.73%. The improvements of the best performer (*MRCARRS*) over other hedging models are reported in Table 3. *MRCARRS* has a largest improvement of 5.70% for the case of *MRCAR* and has a smallest improvement of 0.48% for the case of *PRCARRS*. *MRCARRS* has the best in-sample hedging performance for coffee, sugar and platinum. As for wheat and corn data, the best in-sample performer is *PRCARRS*. *MRCAR* is the best performer for the case of palladium. In general, we find that a regime switching time-varying cross hedging strategy exhibits superior in-sample hedging effectiveness.

Since active hedgers are likely to be more concerned about the out-of-sample future hedging performance, out-of-sample hedging exercises are also performed to justify the superiority of regime-switching time-varying cross hedging models. We find that *MOLS* is superior to *OLS* for wheat, coffee, sugar and palladium but inferior to *OLS* for corn and platinum. If we allow the hedge ratio to be time-varying and use *MRCAR* hedging strategy, we find that *MRCAR* is superior to static hedging for corn, coffee, sugar and platinum but not for wheat and palladium. *MRCAR* has the worst out-of-sample performance for the case of wheat. Allowing the hedge ratio to be time-varying does not always improve the hedging performance. If we further take account of the regime switching effect and adopt the *MRCARRS* hedging, *MRCARRS* is the best performer for corn, coffee, platinum and palladium and is only second to *PRCARRS* for sugar. *MRCARRS* does not show superior performance in the case of wheat. However, for the case of wheat, *PRCARRS* is the best performer. Overall, we find that regime switching time-varying model with multiple futures contracts (either *MRCARRS* or *PRCARRS*) exhibits superior out-of-sample hedging performance.[3]

[3] The correlation between crude oil futures and the underlying wheat, corn, coffee, sugar, platinum and palladium are equal to 0.118, 0.175, 0.120, 0.222, 0.260 and 0.151, respectively. The percentage variance reductions estimated with *MRCARRS* for wheat, corn, coffee, sugar, platinum and palladium are equal to 74.51%, 85.50%, 90.02%, 95.22%, 90.00% and 88.11%, respectively. Higher correlation might have a tendency for higher percentage variance reductions but the relationship is not monotonic. In

The economic value of the proposed regime switching time-varying hedging strategy is also justified by calculating the utility gains. Following other empirical studies (Kroner and Sultan, 1993; Lafuente and Novales, 2003; Alizadeh and Nomikos, 2004; Lee et al., 2006, 2010) we assume that a hedger has an expected utility function given by equation (38) with degree of risk aversion $\kappa = 4$. Table 3 reports the hedged portfolio returns for each hedging strategy and the utility gains of best performer over alternative models. Again, after taking account of the hedged portfolio return, *MRCARRS* has the highest utility for the case of corn, coffee, platinum, and palladium and *PRCARRS* has the highest utility for the case of wheat and sugar. This is consistent with the results should we use the performance measurement of percentage variance reduction. Hedgers will find hedging benefits by adopting regime-switching time-varying multiple futures hedging strategies.

To further take into account of transaction costs, following the convention in the line of hedging literature, we discuss the effect of

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fact, hedge ratio is a complex function of conditional correlation, conditional volatility and predicted regime probabilities and can be shown as

$$\beta^*_{t|t-1} = P_{1,t|t-1} \times \frac{Cov_{s_t=1}(r_O, r_S) Var_{s_t=1}(r_F) - Cov_{s_t=1}(r_F, r_S) Cov_{s_t=1}(r_O, r_F)}{Var_{s_t=1}(r_O) Var_{s_t=1}(r_F) - [Cov_{s_t=1}(r_O, r_{F2})]^2} +$$

$$\left(1 - P_{1,t|t-1}\right) \times \frac{Cov_{s_t=2}(r_O, r_S) Var_{s_t=2}(r_F) - Cov_{s_t=2}(r_F, r_S) Cov_{s_t=2}(r_O, r_F)}{Var_{s_t=2}(r_O) Var_{s_t=2}(r_F) - [Cov_{s_t=2}(r_O, r_{F2})]^2}$$

where $P_{1,t|t-1}$, r_O, r_S and r_F are respectively the predicted regime probabilities, returns on crude oil futures, underlying commodity spot and corresponded commodity futures. The hedging performance is not determined solely by the correlation and whether a model has superior hedging performance is an empirically issue.

transaction costs on hedging performance with utility gains. In Table 3, taking corn for instance, the average weekly out-of-sample returns from hedge portfolio for *OLS* and *MRCARRS* hedging are respectively equal to 0.054 and 0.037 and the average weekly out-of-sample variance of the returns from hedge portfolio for *OLS* and *MRCARRS* hedging are respectively equal to 3.862 and 3.746. Based on equation (38), if an investor uses the *OLS* method for hedging, he obtains an average weekly utility of $U(R_{p,t}) = 0.054 - 4(3.862) = -15.394$. With *MRCARRS*, the investor obtains an average weekly utility of $U(R_{p,t}) = 0.037 - 4(3.746) = -14.947$. Compared with the static *OLS* hedging, the time-varying hedging strategy *MRCARRS* requires frequent portfolio rebalancing. Taking account of the transaction cost y from portfolio rebalancing, the hedger's net benefit from using *MRCARRS* hedging over *OLS* hedging is $0.446 - \mathrm{y}$. This implies that *MRCARRS* hedging strategy is preferred to the *OLS* hedging strategy if $\mathrm{y} < 0.446$ (in percentage). Since the typical round trip transaction cost is around 0.02% to 0.04% (Lee et. al., 2006) for one futures contract (use only corresponded non-energy commodity futures) and therefore around 0.04% to 0.08% for two futures contracts (use both energy futures and corresponded non-energy commodity futures), the transaction costs are far less than 0.446%. As a consequence, an investor with mean-variance expected utility function would benefit from using *MRCARRS* hedging method, even after taking transaction costs into consideration. As shown in Table 3, the best *RCARRS* model (either full switching *MRCARRS* or partial switching *PRCARRS*) has a utility gain larger than 0.08% over both static *OLS* and *MOLS* hedging strategies. This shows that state-dependent dynamic hedging models outperform static models even after taking into account of transaction costs.

To test the statistical significance of the superiority of the regime-switching time-varying multiple futures hedging strategies, we perform Diebold and Mariano (1995) and West (1996) (*DMW*) test with adjusted critical values reported by McCracken (2007) shown in Table 4. We show that static multiple futures hedging (*MOLS*) statistically create hedging gains compared to static single futures hedging (*OLS*) for wheat and coffee at the 5% level. Although *MOLS* is inferior to *OLS* for corn and platinum, the *DMW* statistic is insignificant at conventional level. Overall, the hedging performance of *MOLS* is no worse than *OLS*. The best out-of-sample performers are *PRCARRS* for wheat and sugar and *MRCARRS* for corn, coffee, platinum and palladium. Although not all test statistics are significant, all of them are positive. All test statistics are positive indicating that *MRCARRS* and *PRCARRS* have a tendency to outperform other hedging models.[4]

Figures 1 through 5 exhibit various characteristics of the alternative estimated models. To save space, we only illustrate figures for wheat. Figure 1 and figure 2 compare the hedge ratios of *MOLS* and *PRCARRS* for wheat futures and crude oil futures, respectively. The conditional hedge ratios estimated from *PRCARRS* are very volatile and this shows the necessity of rebalancing hedged portfolio with dynamic hedging strategies to minimize the risk of hedged portfolio. Figure 3 and figure 4 show respectively the hedge ratios of *MOLS* and *MRCARRS* for wheat futures and crude oil futures. The hedge ratios

4 Adding other safe haven and hedging asset, say gold futures, might further improve the hedging performance. We leave this for future study and we thank an anonymous reviewer for this constructive suggestion.

estimated with *MRCARRS* is ranged from 0.321 to 1.322 for corresponded non-energy commodity futures and is ranged from −0.151 to 0.189 for crude oil futures. The range for the hedge ratio of corresponded non-energy commodity futures is equal to 1.001 and the range for the hedge ratio of crude oil futures is equal to 0.340. This is consistent with the finding that the volatility in the transition equations of corresponded non-energy commodity futures is higher than that of crude oil futures. Figure 5 shows the *MRCARRS* estimates of hedge ratios which fluctuate between 0 and 1.

V. Conclusions

The focus of this article has been investigating the cross hedging effectiveness of crude oil futures for non-energy commodity holdings with multiple random coefficient autoregressive Markov regime switching models. We consider both the full switching multiple random coefficient autoregressive model (*MRCARRS*) and partial switching multiple random coefficient autoregressive model (*PRCARRS*) for simultaneously estimating the optimal hedge ratios of crude oil futures and non-energy commodity futures. *MRCARRS* and *PRCARRS* are more parsimonious than trivariate regime switching GARCH models in constructing the regime-switching time-varying multiple futures hedging strategies. We attempt to investigate if multiple futures cross hedging strategy is superior to conventional single futures hedging strategy? We also investigate if further taking account of the regime switching effect improves the hedging effectiveness of multiple futures cross hedging strategy.

Empirical results reveal that in general, multiple futures cross hedging strategy is superior to single futures hedging strategy both in- and out-of sample. According to the Diebold, Mariano and West (DMW) test statistics, the hedging performance of the multiple futures ordinary least square hedging strategy (*MOLS*) is statistically no worse than the single futures ordinary least square hedging strategy (*OLS*). This justifies the superiority of multiple futures hedging over single futures hedging. Results also show that *MRCARRS* is the best in-sample hedging strategy for coffee, sugar and platinum and *PRCARRS* is the best in-sample performers for wheat and corn. Out-of-sample, the best performers are *MRCARRS* for corn, coffee, platinum and palladium and *PRCARRS* for wheat and sugar. Generally speaking, either *MRCARRS* or *PRCARRS* is the best performer for all commodities considered. All *DMW* statistics are positive for the best performer (*MRCARRS* or *PRCARRS*) over competing hedging strategies indicating that multivariate state-dependent *RCARRS* models have a tendency to outperform state-dependent and static hedging models.

Table 1　Summary of the Estimation Procedure of *MRCARRS*

MRCARRS Model	$R_{c,t}=\alpha_{s_t}+\beta_t R_{f,t}+\gamma_t R_{o,t}+\varepsilon_{t,s_t}$ $(\beta_t-\overline{\beta})=\phi_{\beta,s_t}(\beta_{t-1}-\overline{\beta})+v_{t,s_t}$ $(\gamma_t-\overline{\gamma})=\phi_{\gamma,s_t}(\gamma_{t-1}-\overline{\gamma})+u_{t,s_t}$
replace β_t with $\delta_t=\beta_t-\overline{\beta}$ and γ_t with $\tau_t=\gamma_t-\overline{\gamma}$	$R_{c,t}=w_{s_t}+\mathbf{R}_{F,t}\boldsymbol{\delta}_t+\varepsilon_{t,s_t}$ $\delta_t=\phi_{\beta,s_t}\delta_{t-1}+v_{t,s_t}$ $\tau_t=\phi_{\gamma,s_t}\tau_{t-1}+u_{t,s_t}$ $w_{s_t}=\alpha_{s_t}+\overline{\beta}R_{f,t}+\overline{\gamma}R_{o,t}$
Run Kim's Filter to extract δ_t^j and τ_t^j	1. Run Kalman Filter 2. Run Hamilton Filter 3. Approximation
Parameter Estimation	Maximizing the log likelihood function in equation (27) with respect to the unknown parameters $\mathbf{\Theta}=\{p_0,\ q_0,\ \alpha_{s_t},\ \sigma^2_{\varepsilon,s_t},\ \phi_{\beta,s_t},\ \overline{\beta},\ \sigma^2_{v,s_t},\ \phi_{\gamma,s_t},\ \overline{\gamma},\ \sigma^2_{u,s_t}\}$
Recover β_t^j and γ_t^j	$\hat{\beta}^j_{t\|t}=\hat{\delta}^j_{t\|t}+\hat{\overline{\beta}}$, $j=1,2$　$\hat{\gamma}^j_{t\|t}=\hat{\tau}^j_{t\|t}+\hat{\overline{\gamma}}$, $j=1,2$
Estimates of hedge ratios $\hat{\beta}^*_{t\|t}$ and $\hat{\gamma}^*_{t\|t}$	$\hat{\beta}^*_{t\|t}=\hat{\beta}^1_{t\|t}P(s_t=1\mid\psi_t)+\hat{\beta}^2_{t\|t}P(s_t=2\mid\psi_t)$ $\hat{\gamma}^*_{t\|t}=\hat{\gamma}^1_{t\|t}P(s_t=1\mid\psi_t)+\hat{\gamma}^2_{t\|t}P(s_t=2\mid\psi_t)$
One-step-ahead forecast of hedge ratios $\hat{\beta}^*_{t\|t-1}$ and $\hat{\gamma}^*_{t\|t-1}$	$\hat{\beta}^*_{t\|t-1}=\hat{\beta}^1_{t\|t-1}P(s_{t-1}=1\mid\psi_{t-1})+\hat{\beta}^2_{t\|t-1}P(s_{t-1}=2\mid\psi_{t-1})$ $\hat{\gamma}^*_{t\|t-1}=\hat{\gamma}^1_{t\|t-1}P(s_{t-1}=1\mid\psi_{t-1})+\hat{\gamma}^2_{t\|t}P(s_{t-1}=2\mid\psi_{t-1})$

Table 2 Estimates of Unknown Parameters of Alternative Models Data Period is from January 1996 to December 2008

	Wheat			*Corn*			*Coffee*		
	MRCAR	*PRCARRS*	*MRCARRS*	*MRCAR*	*PRCARRS*	*MRCARRS*	*MRCAR*	*PRCARRS*	*MRCARRS*
	Measurement Equation			*Measurement Equation*			*Measurement Equation*		
p_0		1.222	1.224		2.499	-0.333		3.672	5.244
		(0.295)***[1]	(0.309)***		(0.391)***	(0.580)		(0.557)***	(0.986)***
q_0		1.733	1.767		0.868	1.653		4.387	4.812
		(0.277)***	(0.306)***		(0.473)**	(0.468)***		(0.601)***	(0.781)***
α_1	0.041	0.030	0.056	0.075	-0.384	0.279	-0.041	0.035	-0.214
	(0.087)	(0.138)	(0.085)	(0.068)	(0.314)	(0.048)***	(0.091)	(0.065)	(0.182)
α_2		-0.026	-0.040		0.234	-0.726		-0.163	0.101
		(0.092)	(0.134)		(0.045)***	(0.385)**		(0.236)	(0.062)*
$\sigma_{\varepsilon,1}$	1.999	1.171	1.101	1.454	2.990	0.652	1.920	1.212	3.114
	(0.088)***	(0.102)***	(0.099)***	(0.054)***	(0.274)***	(0.057)***	(0.073)***	(0.068)***	(0.151)***
$\sigma_{\varepsilon,2}$		3.341	3.042		0.639	2.820		3.544	0.934
		(0.216)***	(0.267)***		(0.053)***	(0.363)***		(0.200)***	(0.066)***
	Transition Equation			*Transition Equation*			*Transition Equation*		
$\overline{\beta}$	0.739	0.726	0.733	0.949	0.965	0.994	0.581	0.662	0.729
	(0.028)***	(0.036)***	(0.026)***	(0.023)***	(0.021)***	(0.022)***	(0.023)***	(0.136)***	(0.019)***
$\overline{\gamma}$	-0.004	0.009	0.009	0.005	-0.009	-0.012	0.019	0.016	0.018
	(0.017)	(0.017)	(0.015)	(0.012)	(0.009)	(0.010)	(0.017)	(0.016)	(0.014)
$\phi_{\beta,1}$	0.162	0.921	0.705	-0.003	0.099	0.043	0.011	0.994	1.002
	(0.146)	(0.054)***	(0.170)***	(0.043)	(0.114)	(0.100)	(0.065)	(0.006)***	(0.008)***
$\phi_{\beta,2}$			-0.213			-0.118			0.151
			(0.197)			(0.262)			(0.262)

Table 2 (continued) Estimates of Unknown Parameters of Alternative Models

	Wheat			*Corn*			*Coffee*		
	MRCAR	*PRCARRS*	*MRCARRS*	*MRCAR*	*PRCARRS*	*MRCARRS*	*MRCAR*	*PRCARRS*	*MRCARRS*
	Transition Equation			*Transition Equation*			*Transition Equation*		
$\phi_{\gamma,1}$	-0.032	0.335	0.586	0.252	-0.394	1.144	0.121	-0.058	0.068
	(0.084)	(0.212)*	(0.313)**	(3.401)	(0.860)	(0.922)***	(0.609)	(0.149)	(0.128)
$\phi_{\gamma,2}$			0.969			0.660			0.387
			(0.134)***			(0.153)***			(0.619)
$\sigma_{v,1}$	0.314	0.056	0.123	0.290	0.275	0.189	0.287	0.020	0.028
	(0.043) ***	(0.025)**	(0.066)**	(0.024)***	(0.028)***	(0.066)***	(0.025)***	(0.007)***	(0.023)
$\sigma_{v,2}$			0.324			0.417			0.156
			(0.091)***			(0.086)***			(0.028)***
$\sigma_{u,1}$	0.001	0.000	0.000	0.000	0.023	0.000	0.000	0.072	0.000
	(0.025)	(0.068)	(0.025)	(0.035)	(0.026)	(0.016)	(0.051)	(0.024)***	(0.062)
$\sigma_{u,2}$			0.000			0.166			0.065
			(0.045)			(0.077)***			(0.021)***
LL^2	-1508.59	-1457.07	-1454.76	-1317.62	-1194.69	-1190.16	-1502.73	-1388.97	-1377.93
AIC[3]	3033.18	2938.14	2941.52	2651.24	2413.38	2412.32	3021.46	2801.94	2787.86
BIC[3]	3070.49	2994.10	3016.13	2688.55	2469.34	2486.93	3058.77	2857.90	2862.47

1. Figures in parentheses are standard errors and *, ** and *** indicate significance at the 10% level, 5% level and 1% level, respectively.
2. *LL* stands for the log likelihood
3. The Akaike information criterion (AIC) statistic is defined as $AIC = 2\times k - 2\times \ln(L)$ and the Bayesian information criterion (BIC) is defined as $BIC = -2\times \ln(L) + k\times \ln(N)$, where k is the number of parameters, L is the maximized value of the likelihood function, and N is the sample size.

Table 2 (continued) Estimates of Unknown Parameters of Alternative Models

	Sugar			*Platinum*			*Palladium*		
	MRCAR	*PRCARRS*	*MRCARRS*	*MRCAR*	*PRCARRS*	*MRCARRS*	*MRCAR*	*PRCARRS*	*MRCARRS*
	Measurement Equation			*Measurement Equation*			*Measurement Equation*		
p_0		1.810	1.605		3.100	1.215		3.155	1.135
		(0.286)***[1]	(0.282)***		(0.293)***	(0.347)***		(0.365)***	(0.438)***
q_0		1.675	1.724		1.318	2.909		1.524	2.635
		(0.300)***	(0.324)***		(0.345)***	(0.315)***		(0.516)***	(0.318)***
α_1	0.022	-0.032	0.051	0.066	0.234	-0.007	0.004	-0.088	0.013
	(0.101)	(0.110)	(0.099)	(0.055)	(0.325)	(0.028)	(0.038)	(0.173)	(0.052)
α_2		0.041	-0.098		0.006	0.292		0.029	-0.021
		(0.067)	(0.228)		(0.068)	(0.258)		(0.064)	(0.114)
$\sigma_{\varepsilon,1}$	2.197	3.386	0.969	1.237	3.276	0.646	1.454	3.788	1.032
	(0.097)***	(0.199)***	(0.121)***	(0.041)***	(0.303)***	(0.033)***	(0.066)***	(0.648)***	(0.079)***
$\sigma_{\varepsilon,2}$		0.966	3.204		0.656	2.416		1.042	2.698
		(0.099)***	(0.228)***		(0.035)***	(0.208)***		(0.080)***	(0.301)***
	Transition Equation			*Transition Equation*			*Transition Equation*		
$\overline{\beta}$	0.815	0.861	0.927	0.843	0.890	0.903	0.891	0.887	0.908
	(0.030)***	(0.036)***	(0.032)***	(0.026)***	(0.017)***	(0.016)***	(0.025)***	(0.022)***	(0.017)***
$\overline{\gamma}$	0.035	0.013	0.011	0.023	0.009	0.012	-0.002	-0.004	-0.017
	(0.023)*	(0.015)	(0.018)	(0.01)**	(0.008)	(0.008)*	(0.016)	(0.012)	(0.014)
$\phi_{\beta,1}$	0.460	0.858	0.688	0.232	0.127	-0.174	0.156	0.377	-0.203
	(0.271)**	(0.051)***	(0.185)***	(0.121)**	(0.083)*	(0.056)***	(0.080)**	(0.142)***	(0.104)**
$\phi_{\beta,2}$			0.924			0.807			0.180
			(0.088)***			(0.037)***			(0.183)
$\phi_{\gamma,1}$	0.459	0.033	0.230	0.311	-1.108	1.216	0.481	-0.133	2.597
	(0.368)	(0.144)	(0.190)	(1.142)	(0.185)***	(0.259)***	(0.910)	(0.322)	(1.605)*

Table 2 (continued) Estimates of Unknown Parameters of Alternative Models

	Sugar			*Platinum*			*Palladium*		
	MRCAR	*PRCARRS*	*MRCARRS*	*MRCAR*	*PRCARRS*	*MRCARRS*	*MRCAR*	*PRCARRS*	*MRCARRS*
	Transition Equation			*Transition Equation*			*Transition Equation*		
$\phi_{\gamma,2}$			-0.650			-0.491			0.987
			(0.206)***			(0.214)**			(0.068)***
$\sigma_{v,1}$	0.242	0.111	0.096	0.280	0.176	0.145	0.347	0.226	0.156
	(0.066)***	(0.021)**	(0.028)**	(0.023)***	(0.024)***	(0.029)***	(0.023)***	(0.041)***	(0.025)***
$\sigma_{v,2}$			0.154			0.406			0.536
			(0.050)***			(0.077)***			(0.083)***
$\sigma_{u,1}$	0.116	0.000	0.000	0.000	0.000	0.012	0.023	0.000	0.021
	(0.046)***	(0.037)	(0.032)	(0.033)	(0.014)	(0.017)	(0.062)	(0.026)	(0.012)**
$\sigma_{u,2}$			0.130			0.010			0.000
			(0.103)			(0.045)			(0.049)
LL^2	-1570.30	-1497.36	-1491.04	-1193.71	-1027.43	-1006.96	-1398.25	-1335.27	-1322.04
AIC[3]	3156.60	3018.72	3014.08	2403.42	2078.86	2045.92	2812.50	2694.54	2676.08
BIC[3]	3193.91	3074.68	3088.69	2440.73	2134.82	2120.53	2849.81	2750.50	2750.69

1. Figures in parentheses are standard errors and *, ** and *** indicate significance at the 10% level, 5% level and 1% level, respectively.
2. *LL* stands for the log likelihood.
3. The Akaike information criterion (AIC) statistic is defined as $AIC = 2\times k - 2\times \ln(L)$ and the Bayesian information criterion (BIC) is defined as $BIC = -2\times \ln(L) + k\times \ln(N)$, where k is the number of parameters, L is the maximized value of the likelihood function, and N is the sample size.

Table 3 In- and Out-of-Sample Hedging Effectiveness of *MRCARRS* and Alternative Models. Hedging Period Is from January 2009 to December 2010

	In-Sample			*Out-of-Sample*					
	Variance of Hedged Portfolio Return	Percentage Variance Reduction[1]	Improvement of the Best Performer over Alternatives[2]	Variance of Hedged Portfolio Return	Percentage Variance Reduction[1]	Improvement of the Best Performer over Alternatives[2]	Hedged Portfolio Return	Expected Weekly Utility[3]	Utility Gain of the Best Performer over Alternatives[4]
					Wheat				
Unhedged	16.438			26.010					
OLS	5.665	65.54%	0.80%	6.604	74.61%	0.48%	0.008	-26.406	0.511
MOLS	5.664	65.54%	0.80%	6.583	74.69%	0.41%	0.006	-26.328	0.433
MRCAR	5.625	65.78%	0.56%	6.782	73.92%	1.17%	0.024	-27.106	1.210
PRCARRS	5.533	66.34%		6.478	75.10%		0.016	-25.896	
MRCARRS	5.565	66.14%	0.20%	6.629	74.51%	0.58%	0.014	-26.503	0.607
					Corn				
Unhedged	17.757			25.842					
OLS	3.693	79.20%	0.02%	3.862	85.06%	0.45%	0.054	-15.394	0.446
MOLS	3.691	79.21%	0.01%	3.869	85.03%	0.47%	0.050	-15.425	0.478
MRCAR	3.693	79.20%	0.03%	3.843	85.13%	0.38%	0.047	-15.326	0.379
PRCARRS	3.689	79.23%		3.789	85.34%	0.17%	0.045	-15.110	0.163
MRCARRS	3.723	79.03%	0.19%	3.746	85.50%		0.037	-14.947	
					Coffee				
Unhedged	14.049			11.640					
OLS	6.442	54.15%	4.58%	2.293	80.30%	9.72%	0.280	-8.890	4.340
MOLS	6.437	54.18%	4.55%	2.268	80.51%	9.51%	0.273	-8.800	4.250

Table 3　(continued) In- and Out-of-Sample Hedging Effectiveness of *MRCARRS* and Alternative Models. Hedging Period Is from January 2009 to December 2010

	In-Sample			*Out-of-Sample*					
	Variance of Hedged Portfolio Return	Percentage Variance Reduction[1]	Improvement of the Best Performer over Alternatives[2]	Variance of Hedged Portfolio Return	Percentage Variance Reduction[1]	Improvement of the Best Performer over Alternatives[2]	Hedged Portfolio Return	Expected Weekly Utility[3]	Utility Gain of the Best Performer over Alternatives[4]
				Coffee					
MRCAR	6.600	53.02%	5.70%	1.696	85.43%	4.59%	0.214	-6.571	2.021
PRCARRS	5.866	58.24%	0.48%	1.193	89.75%	0.27%	0.090	-4.682	0.132
MRCARRS	5.798	58.73%		1.162	90.02%		0.096	-4.550	
				Sugar					
Unhedged	20.791			36.351					
OLS	11.460	44.88%	16.72%	8.054	77.84%	17.51%	0.357	-31.860	25.133
MOLS	11.217	46.05%	15.56%	7.972	78.07%	17.29%	0.300	-31.587	24.860
MRCAR	9.935	52.22%	9.39%	1.937	94.67%	0.69%	0.031	-7.718	0.991
PRCARRS	8.264	60.25%	1.35%	1.687	95.36%		0.023	-6.727	
MRCARRS	7.983	61.61%		1.738	95.22%	0.14%	-0.033	-6.984	0.257

1. Percentage variance reductions are calculated as the differences of variance of unhedged position and estimated variances of alterative models over variance of unhedged position multiplied by 100.
2. Improvement of the best performer over other hedging strategies is defined as the differences of the percentage variance reduction of best performer and the percentage variance reduction of alternative models. The best out-of-sample performer is *PRCARRS* for wheat and sugar and is *MRCARRS* for corn, coffee, platinum and palladium.
3. Expected weekly utility is calculated based on equation (38).
4. Utility gains of best performer over other hedging strategies are defined as the differences of the expected utility of the best performer and the expected utilities of alternative models.

Table 3 (continued) In- and Out-of-Sample Hedging Effectiveness of *MRCARRS* and Alternative Models. Hedging Period Is from January 2009 to December 2010

	In-Sample			*Out-of-Sample*					
	Variance of Hedged Portfolio Return	Percentage Variance Reduction[1]	Improvement of the Best Performer over Alternatives[2]	Variance of Hedged Portfolio Return	Percentage Variance Reduction[1]	Improvement of the Best Performer over Alternatives[2]	Hedged Portfolio Return	Expected Weekly Utility[3]	Utility Gain of the Best Performer over Alternatives[4]
				Platinum					
Unhedged	10.0559			13.0460					
OLS	2.7890	72.27%	0.67%	1.4541	88.85%	1.14%	0.143	-5.673	0.544
MOLS	2.7741	72.41%	0.52%	1.4697	88.73%	1.26%	0.132	-5.747	0.619
MRCAR	2.7312	72.84%	0.09%	1.3809	89.42%	0.58%	0.110	-5.413	0.285
PRCARRS	2.8258	71.90%	1.03%	1.3148	89.92%	0.08%	0.093	-5.167	0.038
MRCARRS	2.7219	72.93%		1.3049	90.00%		0.091	-5.129	
				Palladium					
Unhedged	26.6913			26.0367					
OLS	5.9464	77.72%	0.65%	3.1932	87.74%	0.37%	0.179	-12.594	0.399
MOLS	5.9443	77.73%	0.65%	3.1728	87.81%	0.29%	0.184	-12.507	0.313
MRCAR	5.7716	78.38%		3.2293	87.60%	0.51%	0.184	-12.733	0.539
PRCARRS	5.8631	78.03%	0.34%	3.2595	87.48%	0.62%	0.202	-12.836	0.642
MRCARRS	5.8117	78.23%	0.15%	3.0970	88.11%		0.194	-12.194	

1. Percentage variance reductions are calculated as the differences of variance of unhedged position and estimated variances of alterative models over variance of unhedged position multiplied by 100.
2. Improvement of the best performer over other hedging strategies is defined as the differences of the percentage variance reduction of best performer and the percentage variance reduction of alternative models. The best out-of-sample performer is *PRCARRS* for wheat and sugar and is *MRCARRS* for corn, coffee, platinum and palladium.
3. Expected weekly utility is calculated based on equation (38).
4. Utility gains of best performer over other hedging strategies are defined as the differences of the expected utility of the best performer and the expected utilities of alternative models.

Table 4 Diebold-Mariano-West (*DMW*)[2] Test Statistics of No Superiority of Best Performer over Alternative Models. Hedging Period is from January 2009 to December 2010

	Wheat	*Corn*	*Coffee*	*Sugar*	*Platinum*	*Palladium*
MOLS vs. *OLS*	1.816**[3]	-0.468	1.546**	0.432	-0.336	1.076
Best Performer[1] vs. *OLS*	0.609	1.058**	4.239***	3.816***	1.564***	0.905**
Best Performer vs. *MOLS*	0.519	1.085**	4.151***	3.856***	1.600***	0.713*
Best Performer vs. *RCAR*	1.380**	0.953*	3.380***	1.299*	0.788*	1.216**
Best Performer vs. *PRCARRS*		0.457	0.910		0.248	1.089*
Best Performer vs. *MRCARRS*	0.978*			0.473		

1. The best out-of-sample performer is PRCARRS for wheat and sugar and is MRCARRS for corn, coffee, silver and palladium.
2. The formula for DMW statistic is shown in equation (39) with the adjusted critical values for nested models tabulated in McCracken (2007). The *N/R* ratio is 0.154 and the number of parameters to be estimated for MRCAR, PRCARRS and MRCARRS are 8, 12 and 16, respectively. The critical values are tabulated for *N/R*=0.1 and 0.2, and we construct the values for *N/R*=0.154 by interpolation.
3. *, ** and *** indicate significance at the 10% level, 5% level and 1% level, respectively.

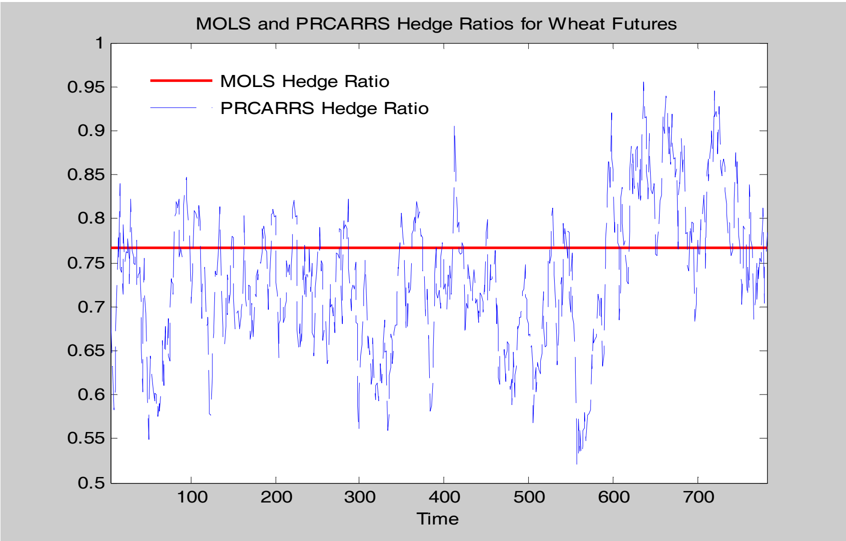

Figure 1 ***MOLS*** **and** ***PRCARRS*** **Hedge Ratios for Wheat Futures**

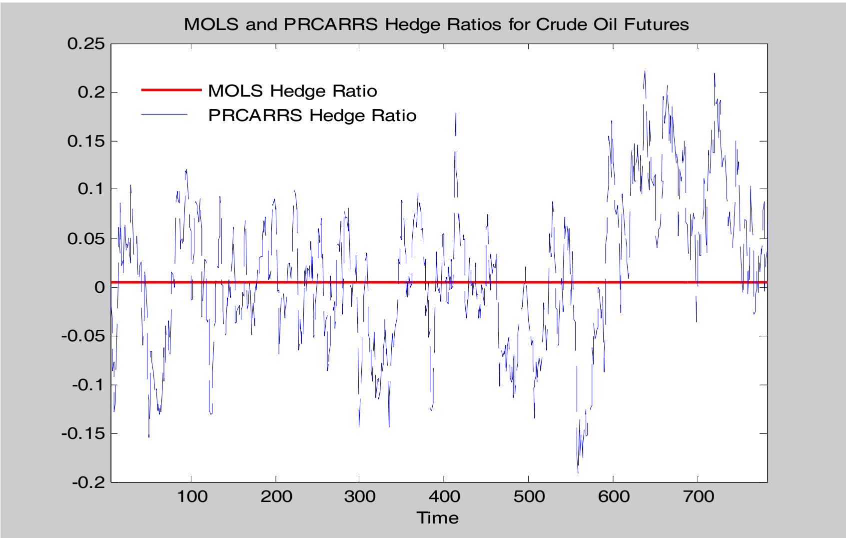

Figure 2 ***MOLS*** **and** ***PRCARRS*** **Hedge Ratios for Crude Oil Futures**

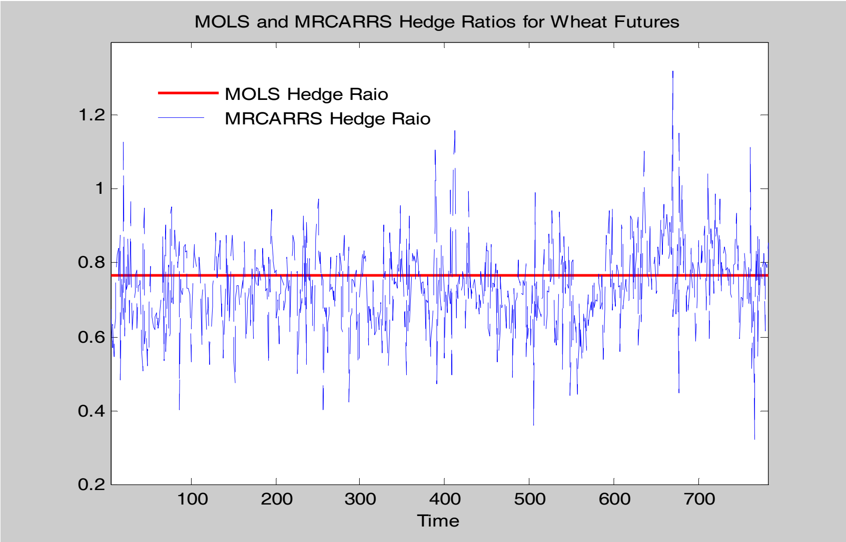

Figure 3　*MOLS* and *MRCARRS* Hedge Ratios for Wheat Futures

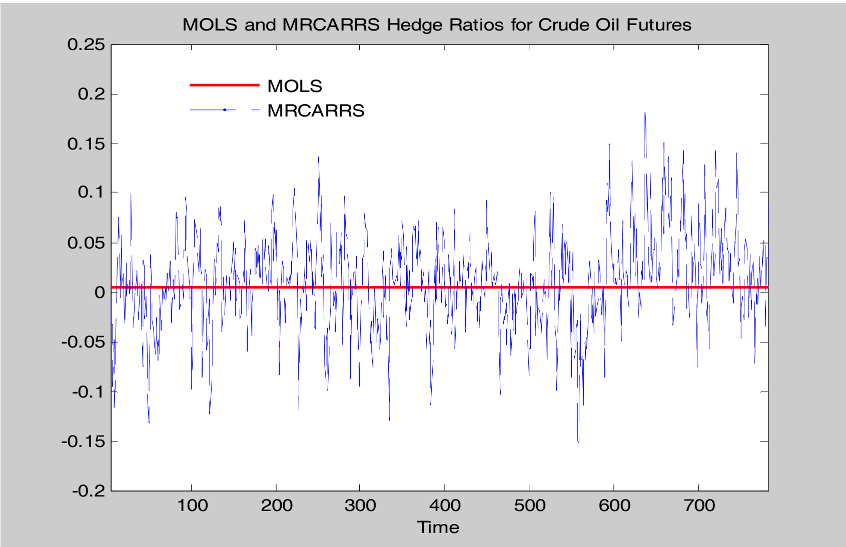

Figure 4　*MOLS* and *MRCARRS* Hedge Ratios for Crude Oil Futures

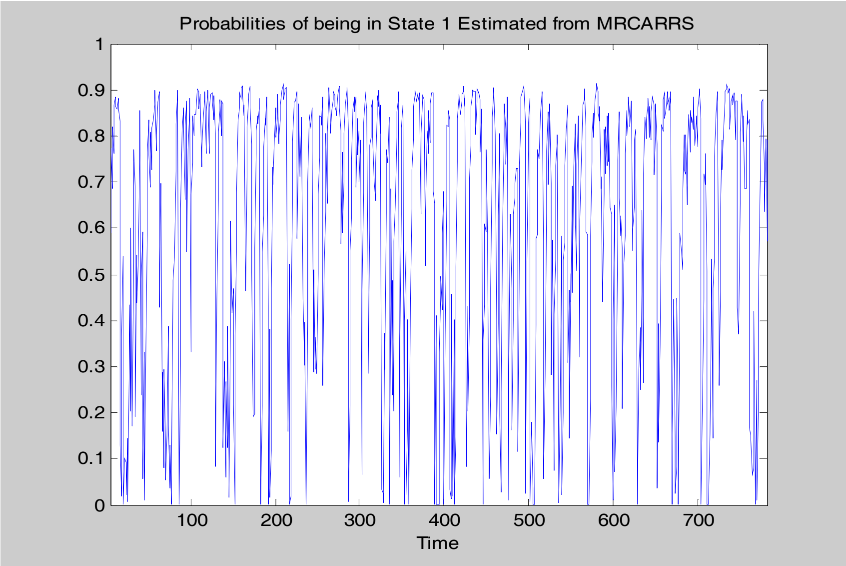

Figure 5　Probabilities of being in State 1 Estimated with *MRCARRS*

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